Chapter 10

1

Rotation Of A Rigid Object About A Fixed Axis

1

Lecture 01

2

Angular Variables

2

Rigid Object

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Analysis models introduced so far cannot be used to analyze all motion.

We can model the motion of an extended object by modeling it as a system of many particles.

The analysis is simplified if the object is assumed to be a **rigid object.**

A rigid object is one that is non-deformable.

The relative locations of all particles making up the object remain constant.

All real objects are deformable to some extent, but the rigid object model is very useful in many situations where the deformation is negligible.

In this chapter another class of analysis models based on the rigid-object model are developed.

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Angular Position

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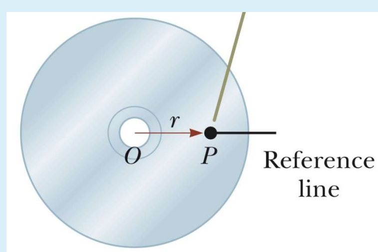
- Axis of rotation is the center of the disc
- Choose a fixed reference line.
- Point P is at a fixed distance r from the origin.
  - A small element of the disc can be modeled as a particle at P.



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Angular Position

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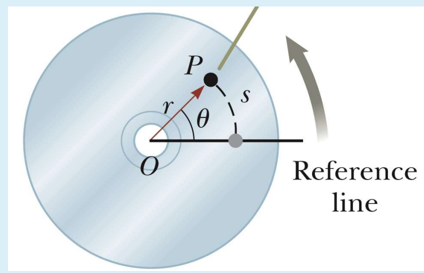
- Polar coordinates are convenient to use to represent the position of P (or any other point).
- P is located at (r, θ) where r is the distance from the origin to P and θ is the measured counterclockwise from the reference line.



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Angular Position, cont.

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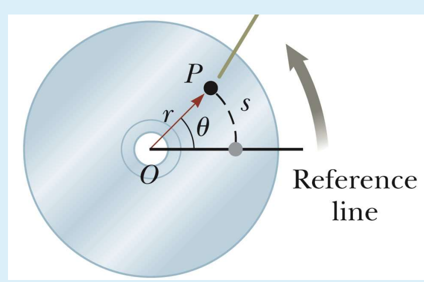
- As the particle moves, the only coordinate that changes is θ.
- As the particle moves through θ, it moves though an arc length s.
- The arc length (s) and (r) are related:

$$s = r\theta$$



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Radian

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This can also be expressed as:

$$\theta = \frac{s}{r}$$

- θ is a pure number, but commonly is given the artificial unit, radian.
- One radian is the angle subtended by an arc length equal to the radius of the arc.
- Whenever using rotational equations, you must use angles expressed in radians.

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Conversions

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Comparing degrees and radians

- $1\ rad = \frac{360^\circ}{2\pi} = 57.3^\circ$

Converting from degrees to radians

- $\theta\ (rad) = \frac{\pi}{180^\circ}\theta(degrees)$

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Angular Position, final

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- We can associate the angle θ with the entire rigid object as well as with an individual particle.
  - Remember every particle on the object rotates through the same angle.
- The **angular position** of the rigid object is the angle θ between the reference line on the object and the fixed reference line in space.
  - The fixed reference line in space is often the x-axis.
- The angle θ plays the same role in rotational motion that the position x does in translational motion.

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Angular Displacement

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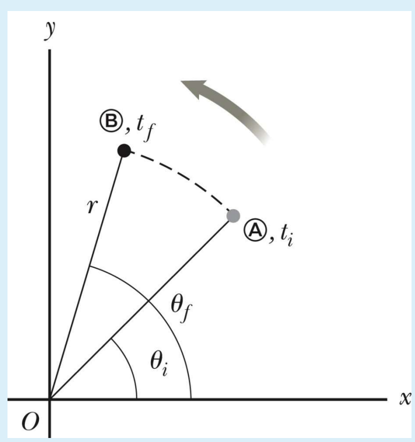
The *angular displacement* is defined as the angle the object rotates through during some time interval.

$$\Delta\theta = \theta_f - \theta_i$$

- This is the angle that the reference line of length r sweeps out.



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Average Angular Speed

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The *average* angular speed, ω_{avg}, of a rotating rigid object is the ratio of the angular displacement to the time interval.

- $\omega_{avg} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta\theta}{\Delta t}$

The *instantaneous* angular speed is defined as the limit of the average speed as the time interval approaches zero.

- $\omega = \frac{d\theta}{dt}$

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Angular Speed

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- This is analogous to translational speed.
- Units of angular speed are **radians/sec**.
  - (rad/s or s^{-1} since radians have no dimensions).
- Angular speed will be positive if θ is increasing (counterclockwise)
- Angular speed will be negative if θ is decreasing (clockwise)

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Angular Acceleration

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The average angular acceleration, α_{avg} , of an object is defined as the ratio of the change in the angular speed to the time it takes for the object to undergo the change.

- $\alpha_{avg} = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{\Delta\omega}{\Delta t}$

The instantaneous angular acceleration is defined as the limit of the average angular acceleration as the time goes to 0.

- $\alpha = \frac{d\omega}{dt}$

Angular Acceleration, cont.

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- Analogous to translational velocity
- Units of angular acceleration are $\boldsymbol{rad/s^2}$ or $\boldsymbol{s^{-2}}$ since radians have no dimensions.
- Angular acceleration will be positive if an object rotating counterclockwise is speeding up.
- Angular acceleration will also be positive if an object rotating clockwise is slowing down.

Angular Motion, General Notes

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- When a rigid object rotates about a fixed axis in a given time interval, every portion on the object rotates through the same angle in a given time interval and has the same angular speed and the same angular acceleration.
  - So: $(\theta, \omega \text{ and } \alpha)$ all characterize the motion of the entire rigid object as well as the individual particles in the object.

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Directions, details

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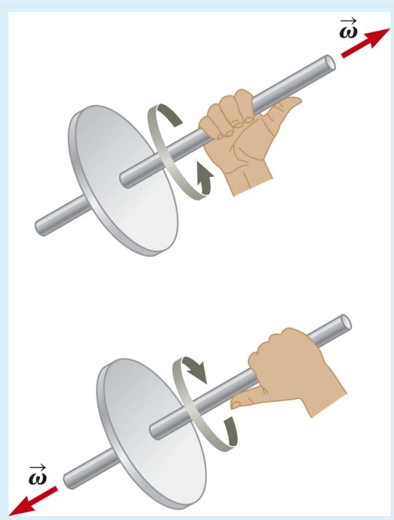
- Strictly speaking, the speed and acceleration $(\omega \text{ and } \alpha)$ are the magnitudes of the velocity and acceleration vectors.
- The directions are actually given by the right-hand rule.



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ω and α derived from θ

Friday, 29 January, 2021 21:38

A reference line in a spinning disk has an angular position given by $\theta = 3t^2 - 12t + 9$, where θ is in radians and t is in seconds.

- Find ω and α as a function of time.
- Find the times when the angular position θ and the angular velocity ω become zero.

Solution: (a) To find ω, we differentiate θ with respect to time:

$$\omega = \frac{d\theta}{dt} = \frac{d}{dt}(3\,t^2 - 12\,t + 9) = (6\,t - 12)\,\text{rad/s}$$

Thus, ω could be negative or positive depending on t. To find the angular acceleration α, we differentiate ω with respect to time:

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}(6\,t - 12) = 6\,\text{rad/s}^2$$

(b) Setting $\theta = 0$, we get:

$$3\,t^2 - 12\,t + 9 = 0 \quad \Rightarrow \quad t = \frac{12 \pm \sqrt{12^2 - 4 \times 3 \times 9}}{2 \times 3} \quad \Rightarrow \quad t = 1\,\text{s and } t = 3\,\text{s}$$

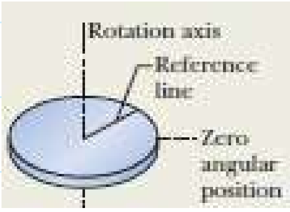
Thus, θ reaches zero at both $t = 1$ s and $t = 3$ s. Setting $\omega = 0$ gives:

$$6\,t - 12 = 0 \quad \Rightarrow \quad t = 2\,\text{s (when } \omega = 0)$$

ω derived from θ

Saturday, 30 January, 2021 16:16

A disk is rotating about its central axis like a merry-go-round. The angular position of a reference line on the disk is given by $\theta = 0.25\, t^2 - 0.6\, t - 1$ with t in seconds, θ in radians, and the zero angular position as indicated in the figure. At what time does θ reach the minimum value? What is that minimum value?



Calculations: The first derivative of $\theta(t)$ is

$$\frac{d\theta}{dt} = -0.600 + 0.500t. \quad (10\text{-}10)$$

Setting this to zero and solving for t give us the time at which $\theta(t)$ is minimum:

$$t_{\min} = 1.20 \text{ s.} \quad \text{(Answer)}$$

To get the minimum value of θ, we next substitute $t_{\min}$ into Eq. 10-9, finding

$$\theta = -1.36 \text{ rad} \approx -77.9°. \quad \text{(Answer)}$$